LARGE WAVES AND BREAKAWAY IN GRAVITATIONAL FILM FLOW

A. E. Bokov and B. G. Ganchev

UDC 532.62

Waves arise when a film of liquid flows over a vertical wall. The wave characteristics do not remain unaltered but vary along the path [1-3].

The wave motion has the character of a weakly stationary ergodic process, and therefore average characteristics can be used to describe it: the average film thickness δ_{av}, the average thickness of the continuous layer δ_c, the average thickness of the ridges δ_r, the average wave heigth $h = \delta_r - \delta_c$, the average frequency ω, and the average wavelength λ.

While the average thickness stabilizes within a short distance of the inlet (as a rule, not more than 200-400 mm), the settling in the wave motion extends to 3-4 m. Small waves of capillary nature are accompanied by large waves, which carry much of the flow [4, 5]. The large waves are covered by capillary ones, and the leading edge of the large wave is steep, with an angle to the direction of motion of about 90°. These waves move with the relatively low frequency ω = 3-7 sec^{-1}.

If the inlet density is high enough, the liquid breaks away from the wave ridges [4, 6]. The first breakaway as individual droplets was observed [4] with an inlet density Γ = 1.3-1.5 kg/m•sec at a distance from the inlet of about 15 m. As the flowrate increases, the breakaway becomes regular and the start of it rises up the flow. The waves cease to grow when a breakaway occurs.

More or less rigorous analytic solutions have been obtained only for the steady-state wave motion in the laminar-wave region, for example in [7, 8]. These solutions cannot be extended to the region of turbulent-wave flow, and they do not describe the changes in wave characteristics with distance traveled or the waves under conditions of breakaway. Some purely empirical relationships for the average wave characteristics [3, 4] do not describe the waves in the region of low inlet densities for Re < 200 or at high inlet densities under breakaway conditions. We give below a simplified model for these large waves, including under breakaway conditions.

We assume that the shape of a large wave can be approximated by the following expression (Fig. 1)

$$\delta = \delta_c(1 + C\bar{l}^n), \tag{1}$$

where $\bar{l} = l/\delta^*$ is the dimensionless coordinate on the wave profile, $0 \leqslant \bar{l} \leqslant \bar{\lambda}$, $\delta^* = (\sigma/\rho g)^{1/2}$ is the length of scale, σ is surface tension, ρ is density, and $\bar{\lambda} = \lambda/\delta^*$ is the dimensionless wave length, which is a function of the path traveled x.

Then $\delta = \delta_c$ for $\bar{l} = 0$ and $\delta = \delta_r$ for $\bar{l} = \bar{\lambda}$.

It follows from (1) that

$$h = \delta_r - \delta_c = C\delta_c\bar{\lambda}^n. \tag{2}$$

We choose the wave profile in such a way as to meet the condition

$$\int_0^\lambda \delta dl = \delta_{av}\lambda, \tag{3}$$

and then

$$\delta_c = \delta_{av} - (1/(n+1))h. \tag{4}$$

The experimental data [3, 4] show that (4) is actually obeyed for n = 4 in the laminar wave region and for n = 5/2 in the turbulent wave one. Parts a-d of Fig. 2 show δ_{av}, δ_c,

Moscow. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 5, pp. 46-51, September-October, 1983. Original article submitted January 26, 1982.

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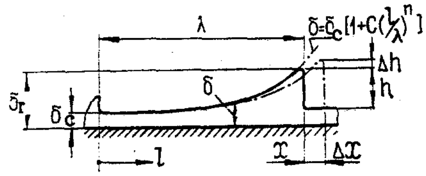

Fig. 1

and h as functions of x (points 1-3 correspondingly) for the following values: $a - \Gamma$ = 0.89 kg/(m•sec), Re = 940, t = 18–28°C b — Γ = 0.493 kg/(m•sec), Re = 675, t = 35°C c — Γ = 0.493 kg/(m•sec), Re = 428, t = 28°C d — Γ = 0.09–0.112 kg/(m•sec), Re = 65, t = 5°C.

We assume that the velocity profile is self-modeling with respect to wave length (Kapitsa used a similar assumption in the theory of capillary waves) and is defined by the quadratic relationship $u = (g/\nu)(\delta y - y^2/2)$ for the laminar-wave state or the power law $u = U(y/\delta)^{1/7}$ in the turbulent-wave one (y is the coordinate along the normal to the wall, ν is the kinematic viscosity, and U is the speed at the surface). Because of the greater thickness of the layer at the wave front, it moves with the speed u_1 greater than the speed u_2 in a depression behind the front. The front thus runs forward into the next depression, and the volume of the wave increases (bulldozer effect). If the self-modeling state in the dimensionless wave profile persists, the height increases, while the thickness of the continuous layer decreases.

In a time $d\tau = dx/u_{c1}$, the volume increment is

$$dV = \left(\int_0^{\delta_c} u_1 dy - \int_0^{\delta_c} u_2 dy \right) dx/u_{c1},$$

and then for the laminar-wave region

$$dV = \frac{1}{2} \frac{\delta_c h}{h + \delta_c/2} dx;$$

and for the turbulent-wave one

$$dV = \frac{7}{8} \delta_c \left[1 - (\delta_c/\delta_r)^{4/7}\right] dx, \tag{5}$$

where u_{c1} is the speed at the wave front at a distance $y = \delta_c$ from the wall.

On the other hand, the increment in the wave volume can be put as

$$dV = \frac{1}{n+1} (\lambda dh + h d\lambda).$$

The volume of the wave is referred to a perimenter of unit length.

We use (1)-(4) to get

$$dV = \frac{1}{n} \left(\frac{h}{C\delta_c}\right)^{1/n} \left[1 + \frac{1}{(n+1)^2} \frac{h}{\delta_c}\right] dh. \tag{6}$$

From (5) and (6) we get for the laminar-wave region that

$$dh = C_l^{1/4} \frac{1}{\delta^*} \frac{\delta_c^{1.25} h^{0.75}}{(2h + \delta_c)(1 + 0{,}04h/\delta_c)} dx,$$

and for the turbulent-wave one

$$dh = \frac{35}{16} C_t^{2/5} \frac{1}{\delta^*} \frac{\delta_c^{7/5}}{h^{2/5}} \left[1 - \left(\frac{\delta_c}{\delta_r}\right)^{4/7}\right] \left(1 + \frac{4}{49} \frac{h}{\delta_r}\right)^{-1}. \tag{7}$$

Comparison with experiment implies that $C_l^{1/4} = 8.345 \cdot 10^{-3}$, $C_t^{2/5} = 3.6685 \cdot 10^{-3}$.

If we assume that the large waves develop from capillary ones, then the data of [5] and our data for $x < 0.1$ m indicate that the initial relative wave height can be taken as approximately constant at $h_0/\delta_{av} \simeq 0.15$.

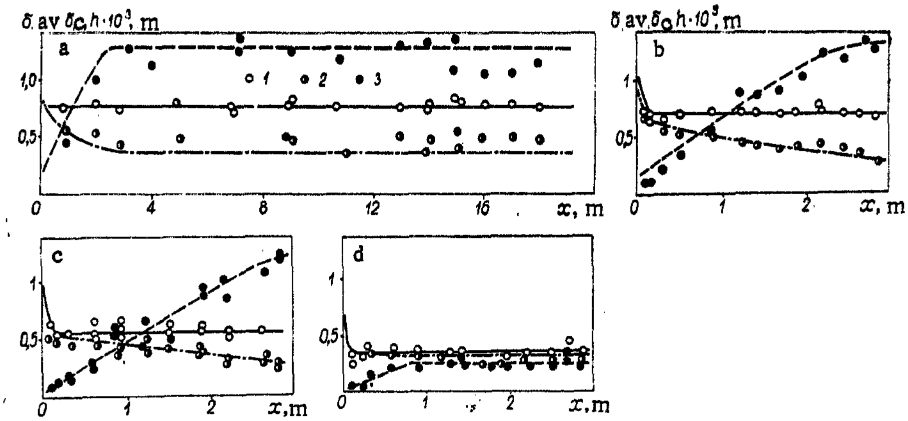

Fig. 2

Figure 2 shows results of some experiments on films flowing on the outer surfaces of cylinders of diameters 60 mm and lengths 3 and 19 m under conditions where there is no breakaway, and values are compared with those calculated from (7). Here δ_{av} was determined from the relationship given in [3].

Calculation from (7) gives a fairly good fit to the experimental results for the development of wave motion; in that part, the wave height increases and the thickness of the continuous layer correspondingly decreases. Then there is a fairly sharp transition to waves with constant average characteristics, and this occurs at a constant value of the Reynolds number Re* for the continuous layer and is independent of the Re found from the total flow rate in the film ($\mathrm{Re} = \Gamma/\rho\nu$).

In the laminar-wave region $\mathrm{Re}_1^* \simeq 45$, and $\mathrm{Re}_2^* \simeq 300$ in the turbulent-wave one; the values of Re* are defined by

$$\mathrm{Re}_1^* = \delta_c^3/(3\nu^2/g), \quad \mathrm{Re}_2^* = [\delta_c/(0.308\,(\nu^2/g)^{1/3})]^{12/7}.$$

Figure 3 shows the variations in δ_c δ_r and δ'_r (without allowance for breakaway) obtained by calculation for the travel of the film for Γ = 1-7 kg/m•sec) for the temperature of 20°C. In the absence of breakaway, δ_c at a given temperature tends to a fixed value, but the values of δ_r differ substantially and increase with Γ (Γ = 1.0; 1.5; 2.0; 3.0; 4.0; 5.0; 7.0 kg/m•sec., curves 1-7 correspondingly).

Under certain conditions, liquid breaks away from the wave ridges, and the results of the experiments of [6] show that this occurs if the kinetic energy of a liquid element in the ridge exceeds the energy at the base by a certain amount and if this energy is sufficient to detach droplets. The critical wave number $\mathrm{We}^* = (\rho(U_1 - u_{c1})^2\delta^*)/\sigma$ is concerned with the breakaway conditions. Within the framework of this model,

$$\mathrm{We}^* = 10^3 \frac{\rho\delta^*}{\sigma\nu^{2/7}} \delta_r^{8/7} (\delta_r^{1/7} - \delta_c^{1/7})^2,$$

and the data on the start of breakaway are represented by $\mathrm{We}^* = 21.5$.

Curve C in Fig. 3 shows the boundary for the start of breakaway in (δ_r, x) coordinates. As the input density increases, there is a substantial increase in δ_r at which the breakaway occurs, then the point at which the breakaway begins shifts towards the inlet. For Re < 1300–1500, breakaway does not occur. Therefore, breakaway occurs only in the region of turbulent wave motion.

We assume that the wave form given by (1) applies after breakaway, while the exponent retains its value (n = 2.5).

Then the increase in the total volume in the wave and beneath it be provided by a volume of liquid proportional to that found from (5):

$$dV_T = k(7/8)\delta_c[1 - (\delta_c/\delta_r)^{4/7}]dx. \tag{8}$$

On the other hand, the increment in the total volume can be put as

$$dV_T = \delta_{av} d\lambda + \lambda d\delta_{av}. \tag{9}$$

We use the condition that We = We* = const, after the start of breakaway, which in our case is equivalent to the condition $\delta_r^{5/7} - \delta_r^{4/7}\delta_c^{1/7} = \mathrm{const}$, together with conditions (1)-(3) to transform (9) to

$$dV_{\mathrm{I}} = -\frac{\delta_{\mathrm{av}}}{n}\frac{\delta^*}{C^{1/n}}\left(\frac{h}{\delta_c}\right)^{1/n}\left[\frac{n+1}{h} - \left(\frac{n+1}{h} + \frac{1}{\delta_c}\right)\frac{n+1}{B} + \frac{n}{\delta_{\mathrm{av}}}\right]d\delta_{\mathrm{av}}, \tag{10}$$

where $B = 1 + \dfrac{\delta_r}{5n\delta_r^{1/7}\delta_c^{6/7} - 4n\delta_c}$; we equate (8) and (10) to get

$$d\delta_{\mathrm{av}} = A\frac{\delta_c^{1,4}}{\delta_{\mathrm{av}}\delta^* h^{0,4}}\left[1 - \left(\frac{\delta_c}{\delta_r}\right)^{4/7}\right]\left[\frac{3,5}{h} - \left(\frac{12,25}{h} + \frac{3,5}{\delta_c}\right)\frac{1}{B} + \frac{2,5}{\delta_c}\right]^{-1}dx. \tag{11}$$

The value of δ_{av} found from (11) under breakaway conditions enables one to determine the proportion of liquid breaking away in the particular cross section:

$$q = 1 - (\delta_{\mathrm{av}}/\delta^0_{\mathrm{av}})^{12/7}, \tag{12}$$

where δ^0_{av} is the average film thickness in the absence of breakaway.

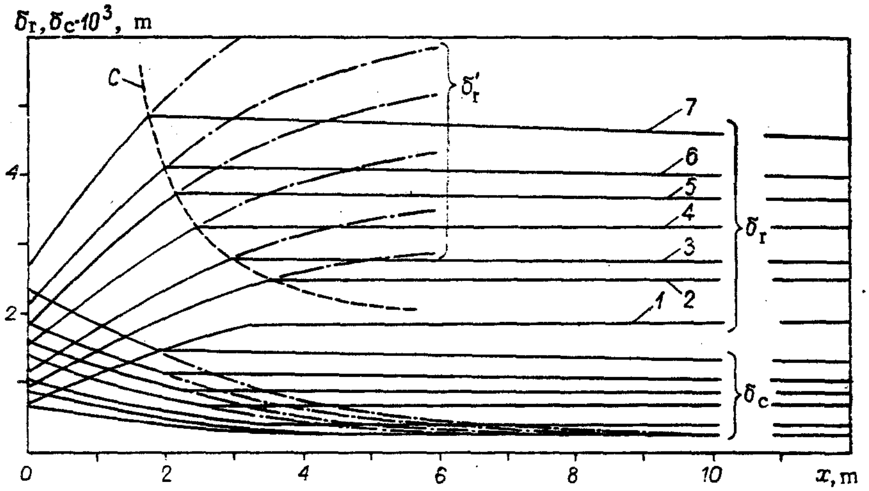

Fig. 3

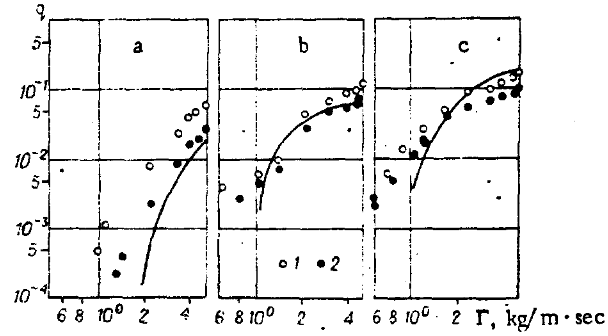

Fig. 4

Equations (11) and (12) with $A = (7/8)kC^{1/n} = 4.375 \cdot 10^{-4}$ give a good fit to the experiments on breakaway. Figure 4 compares the values of q found by calculation and from experiment by sampling with fittings of diameter 40 and 45 mm with the liquid freely flowing within a vertical channel of diameter 54 mm (x = 3, 6, and 16 mm in a-c correspondingly). Calculations confirm the increase in the proportion of breakaway as the path traveled increases and as the irrigation density rises. On the other hand, the rise in the breakaway increment gradient for Γ = 2.5-3.0 kg/m·sec becomes slower as the input density increases.

Figure 3 shows the changes in δ_c and δ_r after the start of breakaway. When breakaway begins, the wave height h and the height of the ridges δ_r no longer increase. There was even a slight fall along with a fall in the average thickness due to the breakaway δ_{av}. At the same time, there is a marked reduction in the rate of fall in the average thickness of the continuous layer.

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AN EXPERIMENTAL STUDY OF INTERNAL SOLITARY WAVES IN A TWO-LAYER LIQUID

V. I. Bukreev and N. V. Gavrilov

UDC 552.593

Experimental data have been derived to check some theoretical results on internal solitary waves in two different cases. In the first, the solitary wave is generated at the interface between two thin layers of immiscible liquids with densities ρ_1 and $\rho_2 > \rho_1$, which are bounded below by a horizontal floor and above by a free surface, and which are at rest in the unperturbed state (Fig. 1a). This situation was first considered theoretically in the Boussinesq approximation in [1], and then under conditions where the Cortvega–de Brees equation applies in [2, 3].

Experimental studies corresponding to this case were performed in [4, 5]. The information obtained there has been supplemented in our studies, in particular in that in the experiments the paths of the liquid particles were recorded along with the speed of the solitary wave and the profile of this. Previously, two laboratory studies had been performed on internal solidary waves in a liquid at rest in the unperturbed state [6, 7], but with a depth distribution of the density different from that given above and with a different flow geometry.

In the second of these cases, the layers were bounded above by an impermeable horizontal cover, and, which is more important, there was a velocity shear between the layers in the unperturbed state (Fig. 1b). It was first predicted that solitary waves can occur at the interface in this case on the basis of the second approximation in shallow-water theory by Ovsyannikov, and this study will be mentioned below as in [8] (a brief abstract of the paper is mentioned in the literature list under this number).

In what follows we use immobile rectangular coordinate system x, y as shown in Fig. 1 (the different positions of the origin along the y axis and the different ways of specifying the depths in schemes a and b have been used for conformity with the corresponding theoretical studies). The deviation of the interface from the equilibrium position is denoted by η,

Novosibirsk. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 5, pp. 51-56, September-October, 1983. Original article submitted July 23, 1982.

0021-8944/83/2405-0652$07.50